

# Harmonic Power Combining of Microwave Solid-State Active Devices

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**Abstract**—A method for combining the power in a selected harmonic of the fundamental frequency for a symmetrical array of oscillating solid-state devices is described and demonstrated. These combiners convert fundamental power to harmonic power with filtering accomplished by symmetry. This technique appears useful for improving the performance of microwave and millimeter-wave power sources by effectively enhancing the frequency-power limitations of existing solid-state devices. An example of the method is provided by a simple three-phase frequency-tripling varactor-tuned transistor oscillator.

## I. INTRODUCTION

POWER COMBINING of solid-state devices at the circuit level has proven to be an important method of achieving relatively high power levels at microwave frequencies for a variety of applications. A variety of novel circuit techniques have been developed and employed in these power combiners both for use with negative-resistance devices [1]–[7] and with three-terminal devices [8]. For a reasonably up-to-date review of these combining approaches, the reader is referred to a recent article by Russell [9].

All of these combiners basically operate to add the fundamental frequency RF power levels available from the individual devices, and hence can be referred to as fundamental combiners. Problems that often arise with such combiners are typically the instabilities associated with the various modes of oscillation which can occur when  $N$  devices are coupled together in a particular network [10]. In particular,  $N$ -way combiners can have  $N$  modes of oscillation, of which only one is desired for power addition, requiring the others to be suppressed. These other modes, however, can be useful for combining power at harmonics of the mode oscillation frequency.

A harmonic power combiner is thus defined as one in which the output is the sum of the powers from the devices at a particular harmonic of the fundamental frequency. In such a combiner, the power in the fundamental and other harmonic frequencies does not appear at the output, but is trapped and converted to the desired output harmonic. In this way, combining efficiency can remain high, depending on the nonlinear properties of the active device. Such combiners can be realized in a straightforward manner in

lossless circuits of radial symmetry.

As pointed out in this paper, harmonic power combining appears to be a useful technique for a variety of reasons. Such a combiner, for example, both combines power from several solid-state devices and performs frequency multiplication, thereby extending the useful frequency range of a particular solid-state device while retaining its power capability. In fact, by using efficient multiplying devices, such as varactors, in combination with active devices, power levels at millimeter wavelengths may be enhanced significantly over those currently available. Furthermore, the symmetry of the combining circuit eliminates the harmonic filters and traps required by single-ended approaches for improved bandwidth capability. Finally, VCO performance at millimeter-wave frequencies should be improved using harmonic combining techniques, since the bandwidth and power limitations will be those associated with the fundamental frequency where better performance is generally available.

In Section II, the basic theory of harmonic power combining is presented, including circuit symmetry requirements, device-circuit interaction, and oscillation conditions, as well as some implications of this combining technique for sources of microwave and millimeter wave length power. In Section III, the harmonic combining approach is applied to VCO applications, and an example of a symmetrical, frequency-tripling transistor VCO is presented to illustrate the basic features of this technique.

## II. BASIC THEORY OF HARMONIC COMBINER OPERATION

### A. Combining Network Description and Properties

A circuit that can be used to combine the power from  $N$  active, negative-resistance devices is shown in Fig. 1. For use in fundamental and harmonic power combiners, the  $N + 1$  port combining circuit is assumed to possess certain symmetry properties. In particular, the admittance ( $Y$ ) matrix of the network will have the form

$$Y_c = \begin{bmatrix} y_{0d} & y_{0d} & y_{0d} & \cdots & y_{0d} \\ y_{0d} & \ddots & & & \\ y_{0d} & & Y_1 & & \\ \vdots & & & \ddots & \\ y_{0d} & & & & y_{0d} \end{bmatrix} \quad (1)$$

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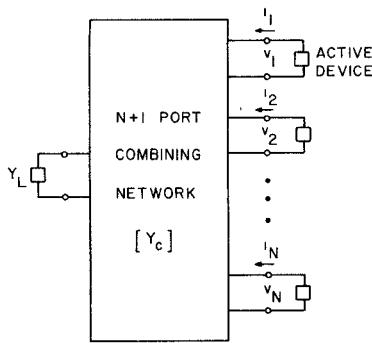


Fig. 1. An  $N+1$  combining network for  $N$  active two-terminal devices.

where  $\mathbf{Y}_1$  has rotational symmetry properties expressed as

$$\mathbf{Y}_1 = [y_{pq}] \quad (2a)$$

$$y_{qp} = y_{pq} \quad (2b)$$

and

$$y_{p+k, q+k} = y_{pq} \quad (2c)$$

where the subscripts are modulo  $N$ . Each row of  $\mathbf{Y}_1$  is thus a “rolled” version of any other row, as would be the case with a radial-symmetric combiner circuit, for example. The properties observed at each device port are identical and each port is indistinguishable from any of the other device ports. Because of the reciprocity and symmetry properties of  $\mathbf{Y}_1$ , there are at most  $N/2+1$  distinct values of the  $y_{pq}$  for  $N$  even and  $(N+1)/2$  distinct values if  $N$  is odd.

If the termination  $Y_L$  on port 0 is absorbed into the network, the result is an  $N$ -port network  $\mathbf{Y} = [y'_{pq}]$  whose elements are given by

$$y'_{pq} = y_{pq} - \frac{y_{0d}^2}{y_{00} + Y_L}. \quad (3)$$

The augmented network clearly obeys the same symmetry conditions as given in (2).

### B. Active Terminations

Ports 1 through  $N$  of the combiner network are assumed to be terminated in active, nonlinear devices. Generally, these devices possess negative resistance over a band of frequencies where power can be extracted in either an oscillator circuit or an amplifier circuit. Of primary interest here is large-signal, steady-state operation of the power combiner when the voltage and current waveforms at port  $n$  will be of the form

$$v_n(t) = \sum_{k=-\infty}^{\infty} V_{nk} e^{jk\omega t} \quad V_{n,-k} = V_{nk}^* \quad (4a)$$

and

$$i_n(t) = \sum_{k=-\infty}^{\infty} I_{nk} e^{jk\omega t} \quad I_{n,-k} = I_{nk}^* \quad (4b)$$

where  $\omega$  is the fundamental frequency. The device nonlinearity will generally result in the multiple harmonics in the

voltage and current waveforms and will establish the interdependencies among the various voltage and current Fourier amplitudes.

In steady-state operation, a harmonic admittance describing function for the nonlinear device may be defined as

$$I_{nk} \triangleq -W_{nk} V_{nk} \quad (5)$$

where  $W_{nk}$  is the describing function<sup>1</sup> whose value depends on the actual operating voltage and current waveforms. For a specified voltage waveform and device nonlinearity, the  $W_{nk}$  can often be determined analytically and will be nonlinear functions of the Fourier voltage components.

If  $\mathbf{V}_k$  is the vector of port voltages at harmonic  $k$ , i.e.,

$$\mathbf{V}_k = \begin{bmatrix} V_{1k} \\ V_{2k} \\ \vdots \\ V_{Nk} \end{bmatrix} \quad (6a)$$

and  $\mathbf{I}_k$  is the vector of port currents at harmonic  $k$

$$\mathbf{I}_k = \begin{bmatrix} I_{1k} \\ I_{2k} \\ \vdots \\ I_{Nk} \end{bmatrix} \quad (6b)$$

then at harmonic  $k$  the terminations impose the constraint that

$$\mathbf{I}_k = -\mathbf{W}_k \mathbf{V}_k \quad (7)$$

where  $\mathbf{W}_k$  is the matrix of admittance describing functions given by

$$\mathbf{W}_k = \text{diag} [W_{1k}, W_{2k}, \dots, W_{Nk}]. \quad (8)$$

### C. Condition for Oscillation

In order to have nontrivial voltage and current waveforms, the voltages and currents at each harmonic must be consistent with the constraints imposed by the combiner circuit. If  $\mathbf{Y}_k$  is the  $y$ -matrix of the combiner at frequency  $k\omega$ , then clearly

$$\mathbf{I}_k = \mathbf{Y}_k \mathbf{V}_k \quad (9)$$

and the condition for steady-state operation is that

$$[\mathbf{Y}_k + \mathbf{W}_k] \mathbf{V}_k = \mathbf{0}, \quad k = 1, 2, 3, \dots \quad (10)$$

where  $\mathbf{0}$  is the empty vector. Nontrivial solutions to (10) at the various harmonics provide oscillatory behavior in the combining circuit. It is noted that (10) actually represents a generally infinite set of coupled nonlinear equations, the solution of which gives the oscillation frequency as well as the voltage and current waveforms. In the case of fundamental power combiners, all harmonic voltage amplitudes<sup>2</sup>

<sup>1</sup>In this case a multiple sinusoidal input describing function [11], [12].

<sup>2</sup>Current amplitudes in the dual case.

except the fundamental are usually assumed zero and (10) applies for  $k=1$  only. The circuit is then designed so that the fundamental powers from the  $N$  devices adds in the load admittance  $Y_L$ .

In the case of harmonic combining, a limited set of higher harmonics is assumed and the circuit is designed to extract only harmonic power in the load admittance. These solutions to (10) for harmonic combiners are considered next.

#### D. $k$ th Harmonic Combining

By writing the condition for oscillation (10) in terms of the network eigenvalues and eigenvectors, the desired solutions for harmonic combining can be found. For the case of a radial symmetric combining circuit as assumed here, the solutions to the eigenvalue equation at frequency  $k\omega$ , i.e.,

$$Y_k \mu_{m_k} = \lambda_{m_k} \mu_{m_k} \quad (11)$$

are given by

$$\mu_{m_k} = \begin{bmatrix} 1 \\ e^{jm_k a} \\ e^{j2m_k a} \\ \vdots \\ e^{j(N-1)m_k a} \end{bmatrix}, \quad m_k = 0, 1, 2, \dots, N-1 \quad (12)$$

where  $\mu_{m_k}$  is the  $m_k$ th eigenvector and

$$\lambda_{m_k} = \sum_{q=1}^N \left( y_{pq} - \frac{y_{0d}^2}{y_{00} + Y_L} \right) e^{jm_k a(q-1)},$$

$$m_k = 0, 1, 2, \dots, N-1 \quad (13)$$

where  $a = 2\pi/N$ . Note that only  $\lambda_0$  depends on  $Y_L$  in (13).

As pointed out by Kurokawa [10], in terms of these eigenvalues and eigenvectors, the oscillation condition (10) can be written as

$$\mu_{m_k}^+ (\lambda_{m_k} \mathbf{I} + \mathbf{W}_k) \mathbf{V}_k = 0, \quad m_k = 0, 1, 2, \dots, N-1,$$

$$k = 1, 2, \dots \quad (14)$$

where  $+$  signifies complex conjugate transpose and  $\mathbf{I}$  is the identity matrix.

Possible solutions to (14) are now easily seen to be

$$\mathbf{V}_k = A_k \mu_{m_k}, \quad m_k = 0, 1, 2, \dots, N-1 \quad (15a)$$

if

$$\lambda_{m_k} + W_{nk} = 0, \quad m_k = 0, 1, 2, \dots, N-1,$$

$$n = 1, 2, 3, \dots, N \quad (15b)$$

for each harmonic, where  $A_k$  is a complex constant. A solution as in (15) can be achieved in the case of identical devices operating with identical voltage waveforms except for a possible time displacement. In this most important

TABLE I  
LOWEST HARMONIC OUTPUT VERSUS MODE FOR VARIOUS  $N$  VALUES

$N \downarrow$	0	1	2	3	4	5	6	7
1	1							
2	1	2						
3	1	3	3					
4	1	4	2	4				
5	1	5	5	5	5			
6	1	6	3	2	3	6		
7	1	7	7	7	7	7	7	
8	1	8	4	8	2	8	4	8

case, if  $\mu_m$  is the eigenvector for  $k=1$ , then clearly

$$\mu_{m_k} = (\mu_m)^k \triangleq \begin{bmatrix} 1 \\ e^{jkma} \\ e^{j2kma} \\ \vdots \\ e^{j(N-1)kma} \end{bmatrix} \quad (16)$$

or  $m_k = mk$ . Since  $mk$  is always an integer,  $\mu_{mk}$  is always a valid eigenvector.<sup>3</sup> Hence for the case of identical devices

$$\mathbf{V}_k = A_k \mu_{m_k}, \quad m = 0, 1, 2, \dots, N-1, \quad k = 1, 2, 3, \dots \quad (17a)$$

$$\lambda_{mk} + W_k = 0 \quad (17b)$$

where  $W_k$  is the  $k$ th harmonic describing function for the identical devices.

Power can now be combined and extracted at the  $k$ th harmonic at the output whenever  $mka = 2\pi l$ ,  $l = 0, 1, 2, \dots$ , or since  $a = 2\pi/N$ , when

$$mk = Nl, \quad l = 0, 1, 2, \dots \quad (18)$$

A fundamental combiner is characterized by  $l=0$ , so that  $m=0$  for all  $k$ . For  $l \neq 0$ , a harmonic combiner with an output at harmonic  $k$  (an integer) occurs for

$$k = \left( \frac{N}{m} \right) l, \quad l = 1, 2, \dots \quad (19)$$

If  $N$  is odd, then output is possible only at multiples of the  $N$ th harmonic of the fundamental frequency. For even  $N$ , harmonic outputs below the  $N$ th are possible (except  $N=2$ ). Generally an output appears whenever  $N/m$  is an integer, for  $m=1, 2, \dots, N-1$ . Shown in Table I are the harmonic outputs possible for the various modes as a function of the number of devices  $N$ .

#### E. Efficiency and Implications of Harmonic Combining

If the combiner circuit is lossless, all the eigenvalues are imaginary except for the  $m=0$  case. Hence, for a  $k$ th

<sup>3</sup> $mk$  modulo  $N$ .

harmonic combiner with  $k > 1$  in (19), the effective terminations for the active devices at the fundamental and some other harmonics are lossless, and these harmonics can be thought of as "idle" which may be used to enhance the combiner performance. The output of the  $k$ th harmonic combiner depends on how effective the energy in the idlers can be transferred to the  $k$ th harmonic for the particular active device used in the network. This represents a form of multiplier design in which the terminations are optimized for the desired harmonic output. Each device will have its fundamental limitations in this regard. Enhanced efficiency may be possible by including a varactor diode in combination with the device to form an equivalent nonlinear device as shown in Fig. 2. If a nonlinear description of the active device is available, solutions to (17b) can be investigated for various circuit conditions using an algorithm such as given by Kerr [13] for solution to the harmonic balance problem.

Many devices are bandlimited in fundamental negative resistance, and varactor diodes may be used to basically convert the available fundamental RF power of the device to a higher harmonic more efficiently than otherwise. If the device is effectively a short-circuit at frequencies above the fundamental, then the varactor in Fig. 2 is primarily responsible for converting fundamental power to higher harmonic power. Of course, without any series loss, the efficiency can approach 100 percent. The power and efficiency of such a combiner can be estimated using the multiplication curves of Penfield and Rafuse [14] in combination with the frequency-power limitation of the active device. At a given output frequency, the amount of power achievable is determined by the efficiency of multiplication, the order of multiplication, the number of devices, and the fundamental power of the device. If the order of the harmonic output is equal to the number of combined devices  $N$ , the power output at frequency  $\omega_0$  can be written as

$$P_0(\omega_0) = N \cdot \eta(N, \omega_0) \cdot P_{RF}(\omega_0/N) \quad (20)$$

where  $\eta(N, \omega_0)$  is the times- $N$  multiplier efficiency for output frequency  $\omega_0$  and  $P_{RF}(\omega_0/N)$  is the fundamental power available from the active device.

Assuming the power available from the active device decreases at the square of frequency,<sup>4</sup> then (20) becomes

$$\begin{aligned} P_0(\omega_0) &= \text{const} \times \frac{N^3 \eta(N, \omega_0)}{\omega_0^2} \\ &= (N \cdot \text{const} / \omega_0^2) N^2 \eta(N, \omega_0) \quad (21) \end{aligned}$$

showing that for  $N$  devices, harmonic combining is preferable if  $N^2 \eta(N, \omega_0) > 1$ . If the varactor is the prime source for harmonic frequency conversion, then the efficiency can, in principle, be determined given the nonlinear characteris-

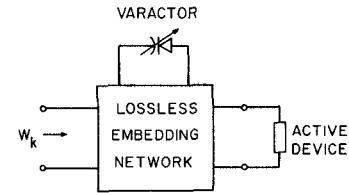


Fig. 2. Equivalent nonlinear device formed by embedding a varactor in combination with the original active device for purposes of enhanced harmonic performance.

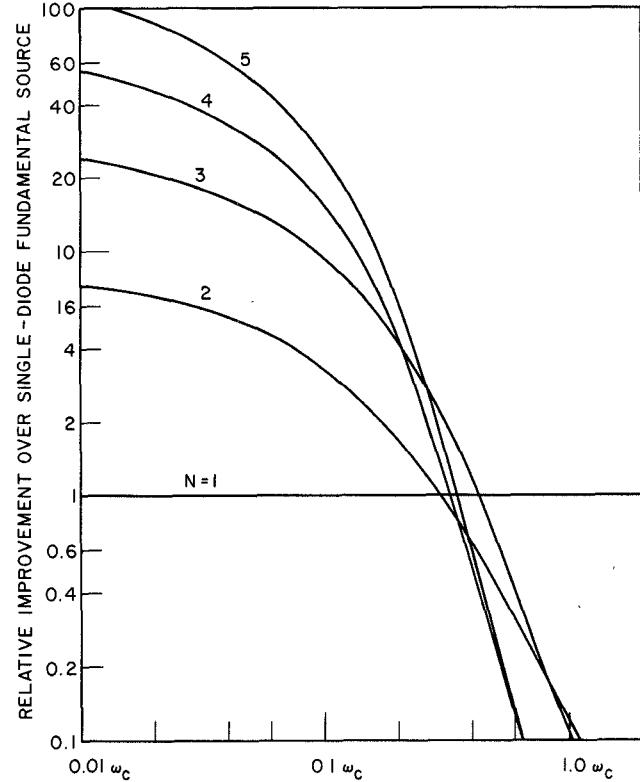


Fig. 3. Relative improvement in power output over a single, fundamental frequency IMPATT diode source provided by symmetrical power-combining frequency multiplication.

tics of the diode. If abrupt-junction varactor diodes and the efficiency curves of Penfield and Rafuse [14] are assumed, the relative improvement in output power over a single device for different levels of multiplication ( $N$ ) as a function of output frequency is shown in Fig. 3. The output frequency is expressed in terms of the varactor cutoff frequency  $\omega_c$ , defined as

$$\omega_c = \frac{S_{\max} - S_{\min}}{R_s} \quad (22)$$

Case  $N=1$  corresponds to a fundamental source. Below approximately 0.3  $\omega_c$  output frequencies, the harmonic combining approach yields higher output power than a single fundamental source. As an example, if  $f_c = \omega_c/2\pi = 1000$  GHz, a source at 100 GHz using a three-diode

<sup>4</sup> $P_{RF} = \text{const} / \omega^2$ .

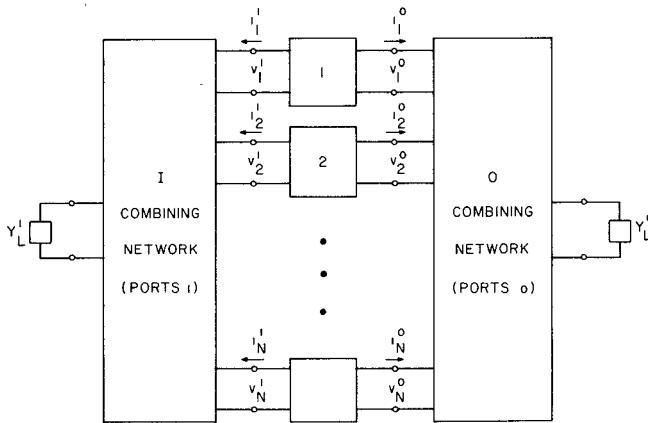


Fig. 4. A possible combining scheme for two-port active networks providing two ports for harmonic power extraction.

symmetrical tripler provides a 10-dB improvement over a single diode fundamental source at 100 GHz. Fundamentally combining ten diodes at 100 GHz for equivalence would be a difficult task indeed!

Another advantage of this type of frequency multiplication over a more conventional approach is that symmetry is used to separate the various frequencies and there is no need for complicated filter designs. In a conventional design, a fundamental source is followed by a multiplier and filters are required to trap the various idler harmonics, generally leading to narrow-band operation. In the symmetrical harmonic combiner, the idlers can be automatically terminated losslessly and no filters are needed. This admits the possibility of broadband performance.

#### F. Three-Terminal Active Devices

The active device in Fig. 1 can be realized from an active conditionally unstable two-port network having an appropriate termination on one of the ports. These  $N$  terminations on the ports for an  $N$ -way combiner can be replaced by an  $N$ -port symmetric circuit similar to that used for original combining. This type of combiner for two-port networks would then be as shown in Fig. 4.

Following similar reasoning as for the one-port active device case (17), the conditions for oscillation, assuming identical devices, are given by

$$V_k^i = A_k^i \mu_{mk}, \quad m = 0, 1, 2, \dots, N-1 \quad (23a)$$

$$V_k^0 = A_k^0 \mu_{mk}, \quad k = 1, 2, 3, \dots \quad (23b)$$

$$\lambda_{mk}^i + W_k^i = 0 \quad (23c)$$

and

$$\lambda_{mk}^0 + W_k^0 = 0 \quad (23d)$$

where the superscripts  $i$  and  $0$  refer to the two device ports. The describing functions (DF)  $W_k^i$  and  $W_k^0$  depend now on the waveforms at *both* ports of the network. As for the one-port case, (18) and (19) still apply so that  $k$ th harmonic output occurs in both the loads  $Y_L^i$  and  $Y_L^0$ . An advantage of this type of circuit is that either of the output ports may

be used as a load while the other is terminated for enhanced performance.

Alternatively, by replacing one of the coupling networks with individual, uncoupled terminations, the  $N$  phases of the fundamental outputs are available at the  $N$  ports. For odd  $N$ , these phases will all be distinct. However, the most appropriate use of  $N$ -phase microwave power seems to be for harmonic generation.

### III. DEGENERATE EIGENVALUE NETWORKS FOR VCO APPLICATIONS

#### A. Combiner Description

A class of harmonic power combining networks useful in VCO applications is shown in Fig. 5. Individual active devices are placed in series with a varactor and resonating element and all such branches are tied together at a common node or combining point to form a radial array. The varactor diodes can be used both for oscillator frequency tuning and for enhanced harmonic generation. If the varactor is included as part of the active nonlinear device, the combiner circuit eigenvalues are particularly simple and are given by

$$\lambda_m = \frac{1}{jX}, \quad m = 1, 2, \dots, N-1 \quad (24a)$$

and

$$\lambda_0 = \frac{1}{jX + NZ_L}, \quad m = 0 \quad (24b)$$

for any frequency. Hence this is referred to as a degenerate eigenvalue combiner since all  $\lambda_m, m > 0$  are identical. For those modes corresponding to  $m \neq 0$ , the combining node appears as a virtual ground.

#### B. Condition for Oscillation

Assuming identical devices, including the varactor diodes, yields the oscillation conditions for this circuit which can be written as

$$jX_k + Z_K = 0, \quad mk/N \neq l \quad (25a)$$

and

$$jX_K + NZ_{Lk} + Z_K = 0, \quad mk/N = l \quad (25b)$$

where  $k$  refers to the harmonic number and  $Z_K$  is the device impedance describing function (DF) defined as

$$V_K = -Z_K I_K \quad (26)$$

for each of the devices at harmonic  $k$ . The impedance DF is used instead of the admittance DF because of the series nature of the combining circuitry.

The solution to (25) is by no means simple, since it represents a generally infinite set of nonlinear algebraic equations. Even if a limited harmonic content is assumed, the solution will typically require numerical computation except for very simple device and varactor nonlinearities. Predictable behavior can, however, be obtained by using measured or calculated device and varactor properties in combination with the appropriate circuitry. What follows is

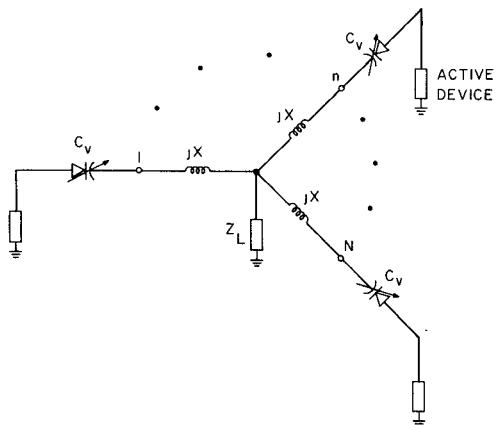


Fig. 5. A simple combiner having degenerate eigenvalues and useful in VCO applications.

an example of a third-harmonic combiner design used primarily to illustrate the basic combining technique.

### C. A Third-Harmonic BJT Combiner VCO

As an example of harmonic power combining with VCO capability, a symmetrical frequency tripling circuit was constructed using the form shown in Fig. 5. In this combiner, the active device used was a bipolar junction transistor (BJT) in the common collector configuration as shown in Fig. 6. With a capacitive termination on the emitter, the negative-resistance properties at the base terminal for the devices used (MRF 901) are typically as shown by the measured data in Fig. 7 for small-signal levels. At the bias levels used, this device has a maximum frequency of oscillation of approximately 3.5 GHz as can be seen by the data.

A first-order design of the third-harmonic combiner is accomplished using the measured device characteristics in combination with an appropriate varactor and series tuning inductance [ $jX$  in (25)]. From (25), the design equations up to the third harmonic are given by ( $N=3$ ,  $m=1$  or 2)

$$j\omega_0 L_t + Z_1 = 0 \quad (27a)$$

$$j2\omega_0L_t + Z_2 = 0 \quad (27b)$$

and

$$j3\omega_0 L_t + 3Z_{L3} + Z_3 = 0 \quad (27c)$$

where  $Z_1$ ,  $Z_2$ , and  $Z_3$  are the DF's for the series combination of the varactor and active device. The design made use of a GaAs hyperabrupt tuning varactor, with the capacitance-voltage characteristics shown in Fig. 8, and a suitable series inductance to provide a small-signal resonance in the 2-3-GHz band at the fundamental frequency. This guarantees oscillator buildup in this frequency band, and large-signal effects and harmonic frequency conversion will result in some third-harmonic output power.

The complete tripling circuit is shown in Fig. 9. Note that the emitters of the three transistors have been coupled together in a delta configuration for simplicity instead of

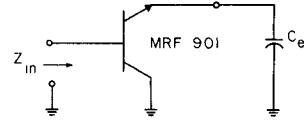


Fig. 6. Common collector configuration for providing a negative-resistance element.

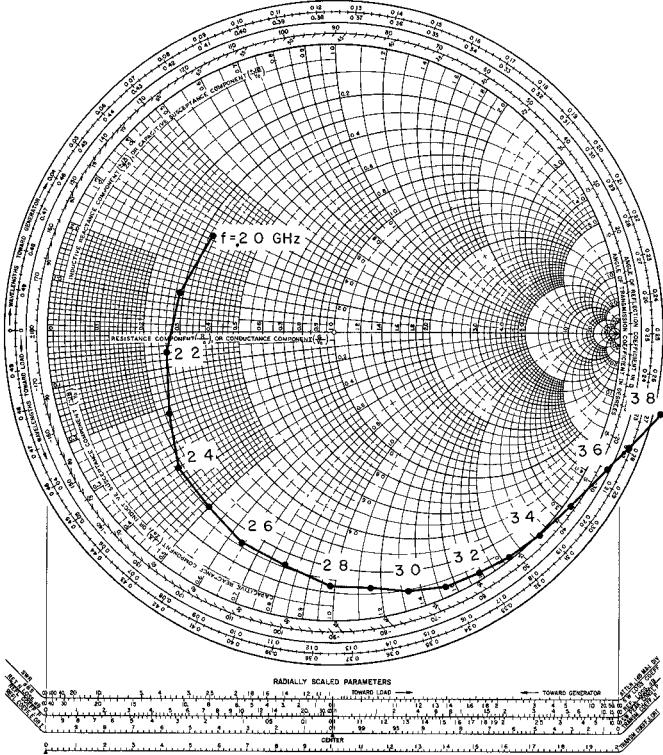


Fig. 7. BJT base impedance (inverted reflection coefficient).

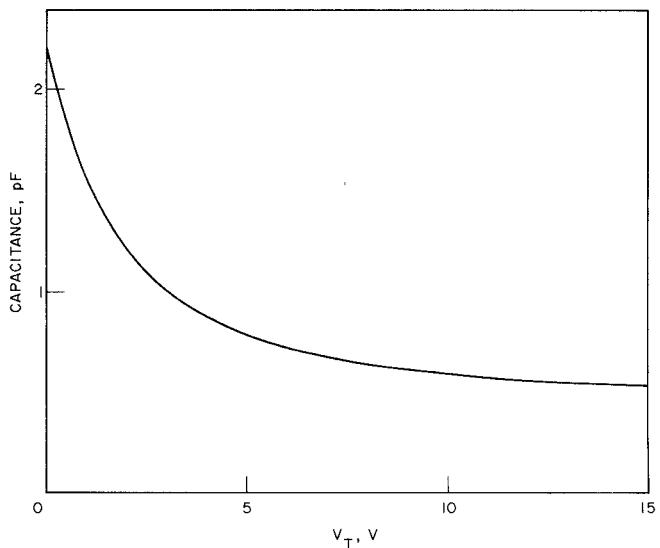


Fig. 8. Capacitance-voltage characteristics of tuning/multiplier varactor.

using three capacitors to ground. Actually, there are three ways in which the capacitive terminations required on the emitter can be realized as indicated in Fig. 10. In the wye configuration (Fig. 10(b)), the capacitance value is the same as required for the single-ended case (Fig. 10(a)).

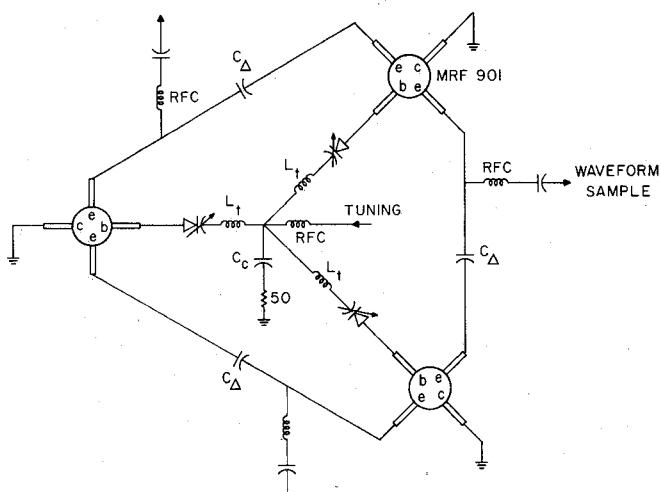


Fig. 9. Circuit used in the third-harmonic combiner (biasing not shown).

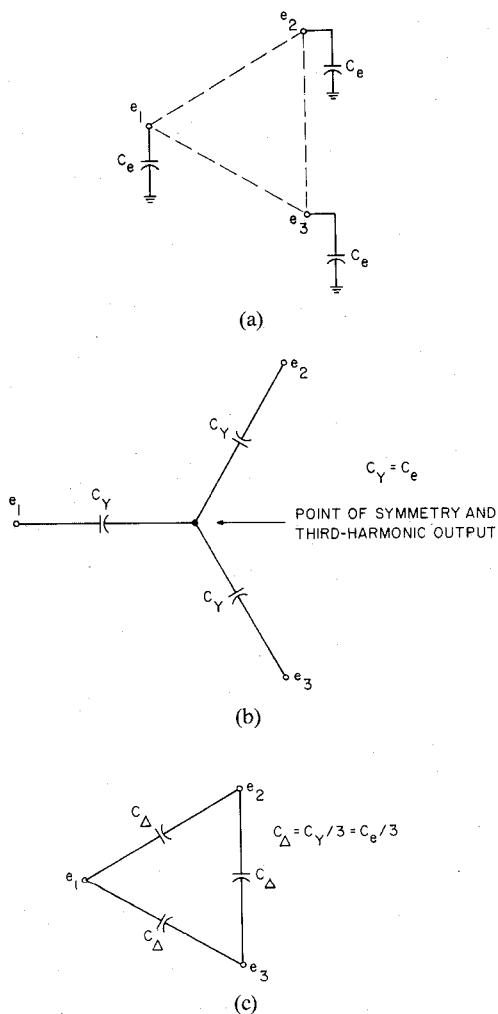


Fig. 10. Three ways of realizing capacitive emitter terminations. (a) Single-ended grounded capacitor. (b) Wye connection for additional third-harmonic output. (c) Delta connection equivalent.

Since the wye center appears as a virtual ground at the fundamental and second harmonic, this configuration provides a second point of symmetry where third-harmonic power can be extracted. Alternatively, this third-harmonic

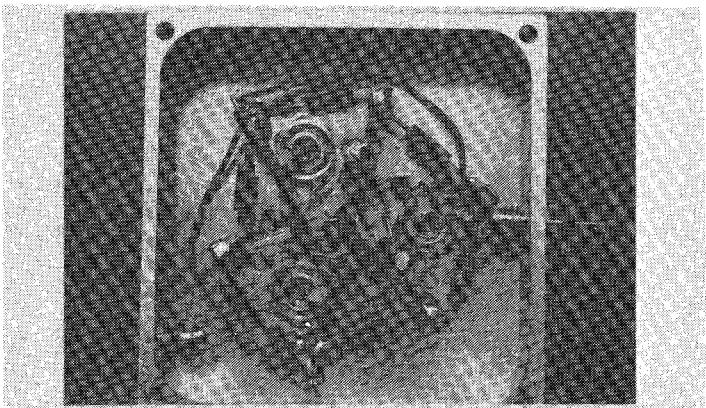


Fig. 11. Photograph of the construction of the three-phase frequency-tripling circuit.

port can be used for lossless tuning of the third harmonic for possibly enhancing performance. Either point of symmetry could be used in this regard. The equivalent delta configuration eliminates the second third-harmonic port and requires  $C_{\Delta} = C_Y/3$  for equivalence. The delta configuration was preferable in this case since it simplifies layout in a planar geometry. The output coupling capacitor was selected to approximately tune out the inductive reactance of  $L_t$  at the third harmonic. This capacitance is approximately the same as that of the tuning varactor since its effective value for each loop is three times smaller. Finally, the bias for all three varactors is provided through a single RFC at the combining point of symmetry. A photograph of the actual circuit is shown in Fig. 11. In addition to the output port, three sampling points were provided at each of the emitters so that the waveforms for each oscillator could be detected.

The performance of the oscillator is summarized in Fig. 12, showing the fundamental, second-harmonic, and third-harmonic outputs at the combining point as a function of frequency (third harmonic) as varied by the varactor tuning voltage. The combiner appears optimally balanced around 7.5 GHz, where the fundamental and second-harmonic levels are down more than 30 dB below the third harmonic. For these transistors, the fundamental power available at 2.5 GHz is typically around 5 mW, so that the 10-mW output at the third harmonic represents reasonable conversion efficiency. Shown in Fig. 13 are the emitter waveforms for the three devices as seen on a sampling oscilloscope. These waveforms were obtained using a two-channel oscilloscope and multiple exposures. All line lengths were carefully matched. The two different displays represent the two modes possible ( $m=1$  or 2) or, in three-phase power systems terminology, the positive and negative sequence. In one case, the time sequence is 1-2-3, while in the other case the sequence is 3-2-1.<sup>5</sup> The reason the waveforms are different in the two cases is that the sampling points couple to each of the three device waveforms, but in differing amounts. Hence, in one case the

<sup>5</sup>The different sequences were obtained by a combination of the turn-on transient and the varactor tuning voltage level.

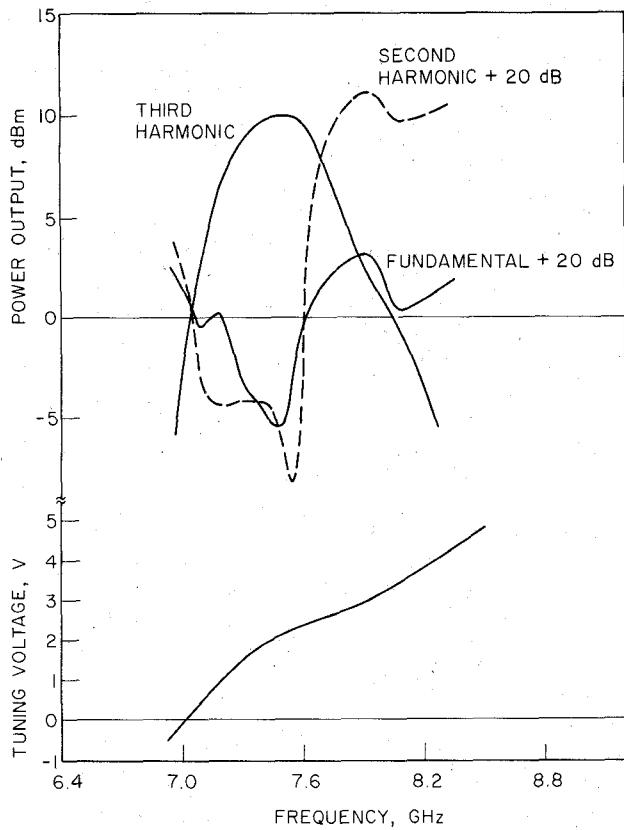


Fig. 12. Performance of symmetrical, three-phase frequency tripler.

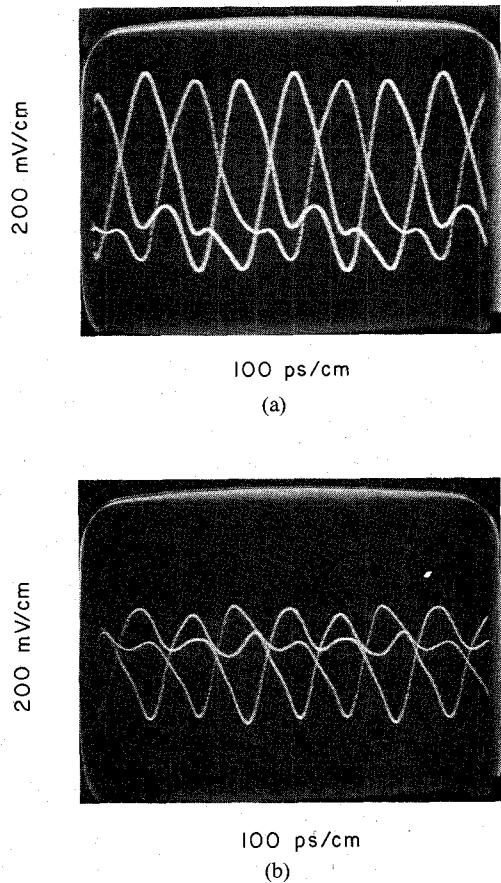


Fig. 13. (a) Waveforms for the 1-2-3 sequence at BJT emitters. (b) Waveforms for the 3-2-1 sequence at BJT emitters.

waveform would be  $af(t) + bf[t - (T/3)] + cf[t - (2T/3)]$ , where  $f(t)$  is the oscillator waveform and  $a$ ,  $b$ , and  $c$  are coupling constants, while in the other sequence, the waveform detected would be  $af(t) + bf[t + (T/3)] + cf[t + 2T/3] = af(t) + bf[t - (2T/3)] + cf[t - (T/3)]$ . If  $a$ ,  $b$ , and  $c$  are all different, the waveforms detected will differ. In either sequence, the oscillator performance was essentially the same. As the frequency was tuned away from the optimum, the  $120^\circ$  phase difference degraded, causing the decrease in third-harmonic output and the increase in fundamental and second-harmonic outputs. It should be mentioned that there was no attempt to match the transistors and they were randomly selected. The varactor diodes were reasonably well matched.

The purpose of the experiment was clearly not to provide power at 7.5 GHz, but to demonstrate the basic harmonic combining technique. It is clear that this technique can extend the effective frequency range of some existing solid-state devices. Furthermore, if good balance can be retained, the VCO limitations of the device can be transferred to a harmonic frequency multiple preserving the percentage tuning bandwidth. This is an important aspect, since it can be shown that fundamental varactor-tuned solid-state oscillators tend to have reduced percentage bandwidths as the operating frequency increases.

#### IV. SUMMARY AND CONCLUSIONS

The purpose of this paper was to identify and demonstrate the technique of harmonic power combining using symmetrical circuits and active microwave solid-state devices. The basic features and theory of operation were established in Section II and an example of a simple three-phase, frequency tripling VCO was presented in Section III. This approach to achieving microwave and millimeter-wave power seems to have several basically useful properties and advantages over fundamental sources as pointed out in the text. These include making use of the mode instabilities occurring in fundamental combiners to advantage, use of symmetry to separate harmonic frequencies, effectively improving the frequency-power limitation of solid-state devices, and simplifying circuit design for comparable performance. Use of this technique in combination with other solid-state devices should also provide enhanced performance of such components as balanced or subharmonically pumped mixers at millimeter wavelengths.

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## Symmetrical Combiner Analysis Using S-Parameters

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**Abstract**—A general theory is developed to predict the potential efficiency ( $\eta$ ) and input impedance ( $Z_{ic}$ ) of symmetrical  $N$ -way combining networks in terms of scattering parameters. A simplified version of the theory, assuming perfect symmetry, is then implemented on a semiautomatic network analyzer (SANA) which is used to characterize 2-way and 16-way  $TM_{010}$  combining networks.

These simplified theoretical assumptions have also been used to predict the degradation effects of power combiners when one or more sources fail. Results indicate that there is room for improvement if proper design techniques are applied.

### I. INTRODUCTION

PRACTICAL REALIZATION of solid-state microwave transmitters are now feasible due to the continuing improvement of solid-state microwave power devices. In applications where power levels of individual microwave

solid-state devices are insufficient to satisfy system requirements, it becomes necessary to efficiently combine many devices to reach the desired power levels. The purpose of this paper is to develop techniques to design, analyze, and characterize efficient solid-state power-combining networks, as well as to present some experimental verification of these techniques. In addition, the possibility of improving "graceful degradation" characteristics will be explored. In general, the theoretical portions of the approaches given here are applicable only to symmetrical power-combining networks.

In the past, solid-state power-combiner design has been implemented by integrating the device matching networks into the power-combining structure. With this approach, it is difficult to isolate problems to either device or combiner and it is also required that alignment to obtain maximum power be done experimentally. The approach presented here will be to separate the total combiner into individual modules. This will 1) simplify analysis, 2) allow alignment

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